'[B]ut this is blog maths and we're free to make up conventions as we go along'

Polymath1 and the Modalities of 'Massively Collaborative Mathematics'

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ABSTRACT

In February, 2009, an international group comprising mathematicians ranging from amateurs to elite professionals converged on the WordPress blog of Cambridge mathematician Timothy Gowers in order to attempt to prove a mathematical theorem—a project Gowers called Polymath1. Their results surprised even the project's most optimistic participants. In six weeks, the group had managed to combinatorially prove the density Hales-Jewett theorem, yielding in the process a host of new mathematical insights.

This paper explores how the mathematicians of Polymath1 worked within and adapted the WordPress blog environment to their uses. I examine from a qualitative sociological perspective how procedural and technical questions interacted in a mathematics research setting as the project moved from its nebulous beginnings toward completion. The paper thus indirectly considers the ways in which such meta-mathematical questions are inscribed in research environments, and opens up several methodological questions for the sociology of mathematics and the Internet. Between the mathematical and meta-mathematical negotiations of the Polymath1 project, there emerges a rich virtual site for the study of collaboration in mathematics and related disciplines.

Categories and Subject Descriptors

K.m [**Computing Milieux**]: Miscellaneous

General Terms

Human Factors

Keywords

Polymath1, blogs, mathematics, online collaboration

WikiSym '10 July 7-9, 2010, Gdańsk, Poland

1. INTRODUCTION

The title of this paper is taken from a perhaps-flippant remark by acclaimed University of Cambridge mathematician Timothy Gowers in a blog post declaring the triumph, less than six weeks from its inception, of a groundbreaking experiment in mathematical collaboration [32].[1] Polymath1 was the first and thus far most successful of what has become a series of experiments in what Gowers termed 'massively collaborative mathematics' [16].[2] The object of these experiments was to perform mathematical research over the Internet in a way which would be as open as possible to participation and would best exploit the strengths of the medium[16, 7, 33]. Gowers envisioned a 'blog maths' which would not merely aim to reproduce its traditional counterpart. Rather, Polymath1 was to test, in the spirit of open experimentation, what might be possible for mathematical collaboration in an age of Internet and information.[3]

From the start, participants and observers in Gowers's project speculated on the broader implications of the undertaking, suggesting that it could be a resource for students and researchers in other areas of science and mathematics as well as a valuable tool even for historians or philosophers [13, 33].[4] This essay offers a first step toward the rigorous social study of Polymath1 by characterizing in a preliminary way the social work performed by and embedded in the project's principal electronic sedimentations. I examine the ways in which the project participants' experiences of their medium of collaboration are recorded and reflected in that very medium, both in the participants' explicit observations and in the clues and traces left in those participants' electronic trails. This study is thus about not just the structure and content of Polymath1 but also the way in which its

[1] Blog entries will be cited with respect to the post's parent thread. Comments will include the date in the comment's timestamp, as well as the number from the Polymath project's internal numbering system (as maintained by Gowers, designated with a '#' symbol), where applicable.

[2] An index of current Polymath experiments is at [31, Main Page]. Citations to the 'Polymath1Wiki' are to the page title.

[3] For a general background on scientific collaboration over the Internet, see [22, 30, 2, 14].

[4] A number of links are collected at [31, Main Page] and in the 'General' column of the table at [31, Timeline]. The project even made it into the *New York Times* annual round-up of the 'Year in Ideas' [44].

modes of practice reveal themselves in the products of those practices. This is thus a study both of Polymath1 and of Polymath1's availability for study.

The case of Polymath1 is especially suitable for such an analysis because the nature of the experiment meant that, in a significant sense for most of its users, its practices and their products were nearly indistinguishable. That is, users of the various Polymath1 blogs and forums engaged with each other by and through the written traces of their collective engagement, most of which remain freely accessible online and are largely unedited. This is not, of course, to say that what I see and experience as a researcher more than half a year after the fact[5] is the same as what Polymath1 participants saw. Posts have been edited, amended, and removed [7, #39, #49, #50, #73, 2–3 February]. The timestamps on comments and threads and the metacommentary of their users show but a shadow of the experience of the project's unfolding, an experience Gowers and Nielsen say at times 'read[s] like a thriller' [13, p. 880].

Viewed after-the-fact, already knowing the project's outcome, it is never fully possible to disentangle the blog text's phenomenal in-the-moment meaning from the distorting effects of its temporally and conceptually removed context of research. Short of 'doing along' the mathematics of Polymath1, many situation-dependent features of blog mathematics are from the start irrecoverable. Even in a contemporaneous context of study, I could never hope to 'do Polymath1' in the way Gowers or other principal contributors did. In this sense, a social study of the mathematical research in Polymath1 is poised to reflect many of the same methodological difficulties and critical possibilities to be found in work on both mathematical research and online communication in general.

Under these circumstances, following the methodological advice of Woolgar and others [43, 18], it will be necessary to deploy a variety of tactics of reflexivity. My account must make itself open to the exigencies of researching Internet phenomena after the fact. Statements about time, sequence, and cause are, here, especially self-indicting, a circumstance I will flag at times with Woolgar's favored device of irony and narrative play. At other times, reflexive gestures will include reflections on my own positions in cultures of both online communication and mathematics research. That I have an undergraduate training in mathematics, that doctoral work in the discipline lies ahead for me, that I studied the history of mathematics at Cambridge while, unbeknownst to me, Gowers began and completed his project at a computer not far from the seminar rooms I occasioned in term, that I would not have had the expertise to contribute much to the project myself, all shade what I do and can know about the course and exigencies of Polymath1. What follows is, in some senses, a conventional study of collaboration on the Internet. In another sense, however, this essay is, like Polymath1, an experiment in how one might write about Internet science and mathematics.

Concerning Polymath1, I will attempt to follow the negotiations between the mathematicians and their blog environment. I shall ask, in particular, in what sense Gowers was really free to make up conventions as he went along. My answer explores this question from two angles. First, I consider the project's guiding narrative, aims, and ideals in the context of its technical medium. Second, I describe the ways in which participants navigated and adapted their blogging medium and how they thematized those negotiations to each other. The juxtaposition of these two lines of inquiry brings into relief the vast and easily overlooked extent to which the project and its medium were co-implicated in one and the same system of mediated exchange, comprising everything from lowly electrical circuits to vaunted scholarly institutions. Gowers cooperated and collaborated with colleagues, institutions, keyboards, and software—actively and tacitly, successfully and unsuccessfully, routinely and intermittently. Through Polymath1, it will be possible to witness with unusual clarity the unfolding of a new space of socio-technical interaction.

2. ORIGIN AND AIMS

Michael Nielsen, according to his website, helped to pioneer the field of quantum computing and is currently writing a book with the working title *The Future of Science* [23].[6] Describing the centuries-old system of scientific reporting as a form of collective memory, Nielsen's book will argue that

> The internet offers us the first major opportunity to improve this collective long-term memory, and to create a collective short-term working memory, a conversational commons for the rapid collaborative development of ideas. The process of scientific discovery—how we do science—will change more over the next 20 years than in the past 300 years. [24]

Nielsen practices what he preaches. His website is a sprawling repository of his thoughts and writings on science, mathematics, computing, and their human dimensions. Its blog format, connected through hyperlinks to both internal and external websites and governed by an ordered array of navigation elements, records highly polished writings and ephemera alike. It holds text written to be read and re-read in the long term and jottings meant primarily for their particular moments of initial intervention. It is a form of memory, one which serves far more than just its author. It is, for instance, an extended part of my memory, a stable online storehouse of almost everything I know about Michael Nielsen.[7]

It is how I remember that on '26 January, 2009 at 10:35 am' (as the timestamp duly reads) Nielsen posted the text of the remarks he had made at a banquet in New Mexico some two weeks prior [25]. I remember Nielsen's speech quite vividly, though I was not at the conference where it was delivered. Nor does it make much difference that I did not encounter Nielsen's blog until late the following summer. The blog, with its text and timestamps transmitted with synaptic speed to my computer screen, tells me all I need to know. Memory has always been a matter of piecing together events from their distant and proximate residues (you, reader, perhaps remember my tapping at a laptop

[5]The present study began in earnest in November of 2009 and focuses on materials available at that time. For a more recent discussion of the project by one of its principal participants, including links to papers reporting its results, see [29].

[6]The working title may have changed to *Reinventing Discovery* since this paper was first submitted [29].

[7]On memory in the sciences, see [4].

keyboard, composing the words you are about to read—this text, like any other, cannot exist without embodying traces of its past production). Titled 'Doing science online,' Nielsen's thoroughly hyperlinked speech begins with a case study from decorated UCLA mathematician Terence Tao's widely-read mathematics blog and then explores the scope and potential of online science and mathematics.

The remarks elicited a flurry of responses ranging from probing to curious to adulatory. At 11:47 am the next day (as the timestamp duly records), thanks to the 'trackback' feature of the WordPress blogging platform both men use,[8] a link was automatically created from Nielsen's blog post to a new remark on the blog of Timothy Gowers (4:47 pm, local time, notes[9] the timestamp) [16]. There, Gowers proposed to test one of Nielsen's central concluding assertions, that future Internet tools will allow scientific collaborations on a much larger scale than had yet been seen.

Timothy Gowers is a distinguished mathematician and a noted pedagogue, highly respected in his field. His blog, as evidenced by the variety of its commenters, is widely read by both professional (and pre-professional) mathematicians and non-mathematicians with an interest in the subject. He certainly had the institutional means to test Nielsen's collaboration thesis. As for his technical circumstances, Gowers and Tao had repeatedly established that their sort of mathematical research and thinking was explicable in a blog format, particularly one like WordPress with a purpose-built capability for displaying mathematical equations.

And while blogs had not been used for the purpose before, it was certainly conceivable that they could support mathematical collaboration. After all, for a group of mathematicians to collaborate all they really need is a way of having a conversation—like the 'Comments' section of a blog post—and a place to write things down, as needed. For the latter, blogs could be seen to outstrip the capabilities of the usual media of blackboards or scraps of paper, for one can easily post images and text alike, and make them searchable and uniformly legible (at least, when the software cooperates). Without a need for laboratories or expensive apparatus, mathematics would seem particularly suited among the sciences to open online collaboration.

Gowers's post spans many topics, from the scholarly justifications for such a project to its potential place in the community of professional mathematicians. He suggests, in particular, that the outcomes of such a collaboration could be reported using a pseudonym such as 'Polymath', a pun suggesting the multifarious efforts of a large group of mathematicians. Blending optimism and skepticism, Gowers expresses great hopes for the potential of large-scale collaboration but points as well to many substantial limitations. He concludes with a touch of pragmatism: "so as an experiment I am going to suggest a problem and see what happens."

The problem Gowers chose was either to find a combinatorial proof of a special case of the density Hales-Jewett theorem, often described for lay audiences as a theorem about a multi-dimensional version of the game 'noughts and crosses' (or 'tic-tac-toe') [20, 11], or to explain why such an approach was unlikely to work [7]. To guide the experiment, Gowers's initial post laid out twelve 'ground rules'. After several days' commentary, including extensive remarks by Tao [16, 28 January] and Nielsen [26], Gowers outlined on his blog a series of background topics [1], elaborated his twelve initial rules into an official list of fifteen [33], and launched the discussion of his chosen problem [7].

3. AN UNUSUAL UNDERTAKING

As readers were quick to point out, many elements of the project had been seen before. Analogies were drawn to open source software development [26], online problem discussion spaces,[10] and research in high-energy physics [16, 30 January (Nielsen)], among a range of both online and offline parallels. Terence Tao and Gil Kalai were among those to suggest that in its broader principles there was little to differentiate Polymath1 from ordinary mathematical research [16, 1 February] [5, #Metacomment.1, 23 February].

It could not be doubted, however, that the project was to be unusual in several respects. For one, Gowers's rules were designed to specifically tailor collaboration around the social and technical affordances of the blog medium to be used. The usual way of going about offline mathematics was explicitly banned. Thus, although the rule was more of a guiding principle than a rigorously adhered-to practice [5, et seq.], Gowers asked participants to adapt even their most routine approaches to mathematical research:

> The guideline, which is designed to stop people going away and doing a lot of work in private—the whole point is that we should all display our thought processes—is that you should not write anything that requires you to go away and do some calculations on a piece of paper.[16]

Rather than the typical litany of reports and reactions that had characterized online mathematics to date, Gowers sought the kind of free-wheeling conversational interplay one finds in interpersonal mathematical collaboration at its best. The proper modes of participation on the websites involved in the experiment, while subjects of negotiation throughout the project, were tentatively specified, referenced, and debated from the start.

Moreover, Polymath1 was explicitly structured to draw in a larger number of participants than would typically work on a single central problem, a deliberate departure from related online collaborations which typically seek to divide work into more-or-less autonomous sub-tasks. In the participants' terms, drawn from computer programming, the work was not intended to be highly parallelizable [41]. The project did not aim merely to solve a mathematical problem. Indeed, while a combinatorial proof was expected to

[8]The website for the blogging platform is `http://wordpress.org/`; for an explanation of the trackback feature, see [39].

[9]I should note here that the timestamp should not be unilaterally trusted. It records only the time of first publication of a post, not of subsequent edits. Moreover, it can be unreliable: Gowers writes on what the datestamp claims to be 17 November but he claims to be the 20th that "Not for the first time in my experience, WordPress's dates have gone funny[...]." [12]. Here, fortunately, the users' power of intervention acts to certify an unreliable datum. Because users expect the timestamp to be accurate and can note when it is not so, one can trust surviving indications with a greater degree of confidence than would be possible if regarding them in isolation.

[10]Tao's comments under [16] refer to his own efforts at [36] and a year-old problem on the 'n-Category Café' blog [21].

have its own rewards, the density Hales-Jewett theorem already had a proof using advanced methods from ergodic theory [1] [7, #3, #4, 1 February]. Rather, Polymath1 was actively engaged as an experiment to test the possibilities of large online collaborations—an experiment for which Gowers's particular mathematical problem was in many ways just the pretence.

These considerations dovetailed with an extraordinary level of reflexivity on the part of the participants. Interjections on the state of the project feature regularly amongst the threaded exchanges on mathematical details. Among the blog posts initiating the project, Gowers devoted an entire page to justifying his choice of problem [41]. Frequent attempts to situate approaches in extant mathematical literature and technical repertoires formed part of a reflexive orientation imported from ordinary mathematical research. For methodological reflection, it was common to seek guidance from analogies to successful non-mathematical enterprises like the user-created Wikipedia [42] or historical episodes in mathematical collaboration such as the canonical early-twentieth-century case of Hardy and Littlewood [28], two Cambridge mathematicians whose long-running litany of joint results and peculiarly formalized way of working together form a common subject for reverence and anecdote in the mathematical community.

Reflexive participation was explicitly invited, as when Gowers interjected that

> I invite others to give their reactions to how things are going for them. My feeling, as I said in the post, is that we are entering uncharted territory (or rather, a different uncharted territory from the initial one) and it is not clear that the same rules should still apply. [5, #Metacomment, 23 February]

Tao led an effort beginning in March to assemble a timeline of significant advances in the project on a wiki Nielsen created some two weeks prior [31, Timeline (and revision history)].[11] One observer even offered reflections about the project's unusual and welcome level of reflexivity [15].

4. THE MEDIUM AND THE MATHEMATICS

4.1 Blog Maths

Polymath1, as I have indicated, centered around a collection of websites. The primary mode of interaction was through the technology of blogging, with Timothy Gowers's personal blog the locus of much of the project's activity. Blogs, of course, are not purpose-built for mathematical research. Emerging out of an early online culture of threaded message boards [6] and sites tailored to share personal journal reflections or news relevant to a particular community, blogs like the ones used in Polymath1 are built around a linearly ordered stream of discrete thoughts or topics, each of which supports its own linear branch of appended comments.

Blogs are particularly distinguished by their temporal ordering and organization. Unlike conventional websites, threaded message boards, wikis, or other online platforms that support user-contribution, blogs are structured almost entirely by the time-order in which contributions are made. In the blog as a whole and within each series of comments, posts are arranged chronologically and are generally not sorted by content, popularity, length, or other criteria. Insofar as Gowers tried to reproduce conversational mathematics, this made blogs a particularly suited medium. Conversations, too, are built around time-ordered sequences of contributions from multiple participants. Of course, its online medium makes blog 'conversations' substantially different from their everyday counterparts, especially with regard to their duration and their (comparatively fewer) limits on participation.

The comparative lack of thematic structure in blogs, as we will see in the next section, means that special efforts must be made to maintain conceptual order over extended blog-exchange. The pages of Polymath1 are littered with both longer summaries of ideas deemed relevant to the ongoing mathematics, as well as micro-summaries and reframings of small sets of ideas either from one place or scattered across multiple threads of discussion. These encapsulations provide a necessary range of touchpoints for readers of a medium where one has few options but to follow its contents in sequence—a sequence which does not necessarily correspond to the reader's interests and goals. In this context, blog users navigate an important order-maintaining dynamic, akin to that of conversation, where the default expectation is that their contributions respond to those in their temporal vicinity. Unlike everyday conversations, however, blog comments have a second default orientation to the parent post under which they are arrayed. Often, these two orientations are maintained simultaneously, and a commenter often begins her or his post with specific textual cues to an alternative referent if that post will break from either of these default orientations.

Participants in Polymath1 were skilled users of the blogging platform. Their posts show in both form and content that a large number of the key participants were active bloggers or had past experience with blogging, and still more had been involved in long-distance mathematical collaborations where email and other Internet communication tools had been essential. There was a strong selection bias in favor of blog-literate mathematicians. It was not enough to have access to the Internet, in no small part because the rapid timeline of the project meant that one was unlikely to become aware of the project in time to participate unless one actively followed one of the many blogs or Internet forums which commented on the formation and progress of the project in its early stages. In my own case, having heard of both Gowers and Tao well before the start of Polymath1 and even having visited Tao's blog in the past, news of the project did not reach me until a conversation at a history of mathematics workshop in July.

A visitor to Gowers's blog, hosted by WordPress.com, is met by a plain blue banner heading with a large white title 'Gowers's Weblog' and, in a much smaller font, its subtly understated subtitle 'Mathematics related discussions.' Be-

[11] Tao announced the Polymath1Wiki's creation at [9, 12 February]. Its purpose was to collect pertinent background information which was no longer part of the active 'foreground' of exchanges on the Polymath1 blog entries. It was thus more of a collaboration resource than a collaboration medium, and does not appear to have been as actively used in support of the ongoing research discussions, themselves, as might have been possible.

neath the heading are two columns. On the left, a larger column displays bold-titled entries, complete with a datestamp, the beginning of the entry's text, a category heading such as 'polymath' or 'Somewhat philosophical,'[12] and a link displaying the current number of comments associated to that entry. The right column contains links to resources related to WordPress and the use of xhtml, followed by a searchbox, a 'Blogroll' of mathematical blogs associated to Gowers's, a short list of 'Mathematical websites,' and a number of tools and links for navigating the various topics and posts on the blog.

Clicking on one of the entries brings the visitor to a page with the same centered heading, followed by the full text of the entry and a list of timestamped comments. Comments appear on the list in one of two ways, and a typical Gowers post, because his blog is widely read and discussed, contains a mix of both forms. Comments could be automatically appended to the post using the 'trackback' feature which posts a link to an entry in another blog along with a short excerpt of that blog entry to encourage visitors to Gowers's site to navigate to the linked post. Alternatively, a visitor can make a comment directly using a form at the bottom of the page. If the visitor is not logged in to a WordPress.com account, the form gives a space for the visitor's name, email address, website, and comment. When logged in, the identifying information is assumed from the account profile, with the website typically taken as the visitor's WordPress blog. In both cases, there is an option to request an email notification of any further comments on that entry. At the top of the page, a logged-in user is also greeted by a toolbar with further account-based navigation tools.

These overt features of the blog platform delimited the Polymath1 project's participants' ways of both accessing and contributing to the mathematical record, as well as their possibilities for modifying the medium to their mathematical and social needs. Gowers, as his own site's administrator, had certain additional privileges to govern the site's appearance and modes of interactivity. This was particularly apparent in two areas: his ability to constrain how comments were added to the discussion and his ability to create new blog entries under which comments could be posted, both of which are discussed in the next section. Users, of course, were not completely beholden to the limitations imposed both deliberately and incidentally by Gowers's blog management. They were free to link from his blog to their own websites or to third party webpages, and often did so. Tao's blog and Nielsen's wiki became prominent supplements to the mathematical exchanges managed by Gowers.

At this point, another feature very much in the minds of Polymath1's participants bears mention. The mathematical work attempted in the project drove many a use and adaptation of the blog environment, but it is equally true that the blog environment constrained and enabled certain kinds of mathematical activity. Gowers aimed for a blog which would facilitate a large scale mathematical conversation, as opposed to the exchange of thought-out writings that are more typical of written mathematical exchanges. He got, unsurprisingly, something in between those two extremes, with comments necessarily bearing more meditation than they would in oral conversation but maintaining some of the tentativeness of that more transitory medium. One goal of this format was to recreate the elements of mathematical creativity endemic to informal exchanges, elements which often slip away as an idea is processed and hewn into a circulable text. Gowers's attempt to discipline the scratchwork of his interlocutors, artificially regulating something which is naturally foreclosed in in-person speech, made it possible for a wider range of ideas to be semi-systematically considered than would be possible in ordinary written (or, for that matter, memory-limited conversational) exchanges.

As for which of those ideas became important to the project, the blog medium favored those that could be passably exposited in the blog itself, first and foremost, as well as the wiki which gathered supporting text as the project grew more complex after its second week. Systematic exploration of bounding values was no doubt facilitated by their ability to be organized in tables, as in [40]. Other mathematical approaches risked faltering due to the inability of contributors to communicate or grasp them sufficiently. Indeed, Gowers acknowledged from the start that his choice of problem would make it impossible for many potential contributors to join the project in the first place [41], and the problem of adapting to different contributor expertise is a recurring theme in the metadiscussion surrounding Polymath1.

4.2 Threading

The structure of the blogging platform means that a comment on a WordPress blog necessarily harbors more than a mere thought or reaction. It is endowed in its very mode of creation with gestures of interconnectivity, filled with often oblique or fragmentary allusions and associations and connected through text and hyperlink to other parts of the page, blog, and Internet. It is, to wit, very much like a memory—albeit a shared memory working in very different registers for different users. This is as it should be: research in mathematics consists of situating and reshaping often highly polysemic claims, whether in the mind of a single mathematician or in the social spaces of a small or 'massive' collaboration, a process Claude Rosental calls de-monstration [35].[13]

Much of the work of blog maths turned out to be in managing those allusions and their different registers. Blog posts functioned as purveyors of mathematical thoughts, but also of discussions surrounding how those thoughts should be marshalled. The latter issue was crucial to the former. Nielsen noted in late March that the Polymath1 project had elicited "More than 1000 mathematical comments" and led to "approximately 59 content pages" on the Polymath1Wiki, with "notable mathematical contributions being made by 23 contributors to date" [27]. At a rate exceeding twenty posts per day, many of which were somewhat longer and more detailed than the short one-thought-at-a-time comments Gowers envisioned, the information pouring into the project could not be assimilated without a corresponding active management of the very medium of collaboration.

Two factors for effectively assimilating ideas seem to have been most salient to commenters on Gowers's initial post [16]. The first, whose urgency faded quickly as the project's substantive mathematics got underway, was the challenge

[12]Due to the design of WordPress, strategically chosen category headings could be used to organize information dispersed across several blogs. See [1, 30 January].

[13]Rosental studies what might be called 'Usenet logic' (as opposed to 'blog maths'), a medium and discipline sharing much in common with those of blog maths, and a significant technological predecessor to blog-based scientific collaboration.

of creating visibility for the most productive ideas. Glossed by analogy to data processing as the problem of 'signal-to-noise,' this factor was largely addressed by the frequent participation and commentary of Gowers and Tao, two luminaries whose ongoing assessments appear actively to have shaped the tone and course of the participants' research to a particularly strong degree.[14] Without their interventions, other collaboratory media, such as the Polymath1-Wiki, would likely have been more important for the task of amplifying the 'signal' of potentially significant mathematical contributions. The second, remaining visible well into the project, was the problem of adapting the non-linear nature of mathematical research and creativity to the at-times-frustratingly linear structure of the blog environment.

User comments reflect an image of the blog as a simultaneously rigid and flexible platform. Thus, Tao remarked in a single comment both that "we are beginning to bump up against the limits of the wordpress environment" and that, though it may be "quite tedious," it would always be possible to reorganize the information on any blog pages which were to be used [33, 2 February]. Tao prioritized maximizing the flow of information onto "the single massive thread" with the promise that "it can be sorted out later." Within the initial research thread, participants began to pick out subsidiary lines of discussion [7, #30, Jason Dyer's 'meta-comment' after #34 (2 February)]. Gowers began labelling his remarks under 'Thread titles,' and later contributors followed his lead [7, #60 (3 February), et seq.] [33, Gowers, 3 February]. After considerable discussion involving comments and a poll spanning at least three different formal entries on Gowers's blog, Gowers elected to conservatively divide the mathematical discussion into streams taking place under multiple entries on his and Tao's blogs [33, 7, 34]. As a result, these procedural discussions were replaced in the comments by efforts at managing when to start a new thread, as well as commentaries on whether such threading was working as intended [9, #Metacomment, 12 February].

Gowers also promoted a system of numbering posts so that later comments could refer specifically to earlier ones [7].[15] This early-established convention created difficulties after the decision had been made to split the research into multiple threads. Employing the engineering protocol of spectrum division, Tao and Gowers elected to start the comment numbers in each new thread at a new multiple of 100 [40], allowing each numbered comment (some, particularly those on procedural matters, were not numbered) to have its own unique number in the project. So that the allotted band of comment numbers would not run out, particularly long discussions were assigned continuation threads, as well [40, #277, 5 February] [5].

Starting in the #800's series of comments, after another discussion and poll [38], Gowers enabled a WordPress feature allowing users to attach comments to other comments so that they no longer formed just a single list below the parent entry [5]. This preserved the general temporal organization of the blog format while increasing its internal division along conceptual lines. The sub-commenting (or 'threaded comments') facility added another layer of internal reference for comments on comments and pulled many 'follow-up' comments out of the normal temporal order and into a smaller temporal sub-order tied to the commented-upon entry. This also changed, sometimes dramatically, the surveyable features of the blog comments page. In particular, it became much easier to pick out rapidly some of the posts which had elicited special attention, based on the length of their sub-comment trails or the presence of a remark by one of the project's more prominent participants.

Indexing could also be achieved by less formal means. Participants, particularly Gowers, announced their intentions to post elaborations and follow-ups [1, 32]. Gowers provided a particularly vivid example of the management of both online and offline contexts and circumstances in such index-work when he announced: "Got it! Details below in an hour's time (after my lecture). It will be comment 812 if nobody else has commented by then" [5, #803.3, 24 February]. In addition to index numbers, comments could be referenced by their authors or subjects [7, #21, 2 February]. The built-in interconnectivity of WordPress blog comments hardly sufficed for the purposes of Polymath1. As the project unfolded, so too did the participants' attempts to supplement a system of commentary decidedly not made for mathematical research. The blogging platform proved amenable to adaptation, but continued to constrain the range of adaptations that could be deployed. In that sense, Tao's image of an obstinate-yet-pliable platform was particularly apt.

4.3 Notation

WordPress forced adaptations in other areas, as well. In any collaboration, online or offline, it is necessary to establish conventions of writing and notation. Gowers's initial series of posts laid out a working system of notational conventions, based on those within the sub-disciplines in which he and many of his fellow collaborators operated, and participation was structured so that new representations could be improvised and rapidly adopted as needed. In particular, the established dynamic meant that commenters were unlikely to develop ideas and their corresponding notation from scratch, and even the division of the research into parallel threads on different webpages left few opportunities for conflicting notations to emerge.

Beyond these, the participants made extensive use of another common resource for representing and configuring mathematical symbols and ideas on the blog. Initially developed by computer scientist Donald Knuth, the TEX typesetting system, and in particular the LATEX variant introduced by Leslie Lamport, has become the dominant standard for producing mathematical publications across a wide range of disciplines and institutional contexts. My own background in mathematics forced me into early contact with the system (a professor insisted I learn to use it, in exasperation with my handwriting), and I am not alone in using LATEX for even my non-mathematical writings, including the present one. Using LATEX, mathematical expressions can be produced using a series of commands controlling what symbols are used and where they are placed. A computer program processes these inputs and gives as an output an electronic document which can be printed or made available online. As a lingua franca for mathematical notation, LATEX markup has also been taken up in a range of peripheral attempts to simplify the process of rendering mathematical expressions

[14] One participant remarked of Gowers and Tao that "it's awfully tough to keep up with a certain pair of powerhouses who post regularly to the project :)" [5, #Metacomment, 23 February].

[15] Tao further suggested identifying comments by author and timestamp [7, #39, 2 February].

in other contexts, particularly on the Web.[16] In my sociological work on practicing mathematicians, I commonly see LaTeX commands in emails and other typed documents, even when they are never intended to be processed by the author or reader into a final typeset form. Conventions from LaTeX and computer programming even emerge, on occasion, in handwritten mathematics, particularly on the chalkboard.

One feature of WordPress which made it particularly attractive to the Polymath1 mathematicians was its ability to process inputs written with LaTeX markup. This blessing, however, was also a curse. Frequent early explanations about how to adapt LaTeX inputs for WordPress indicate that a significant number of Polymath1 participants did not or were not expected to have substantial experience using LaTeX on WordPress blogs [7, #51, 2 February] [10, 6 February]. More telling, however, were the frequent errors and exclamations of bewilderment or dissatisfaction at failed attempts to achieve the desired results on a blog post [7, #2, #5, #51, 1–2 February] [8, #596 et seq., 23-24 February].

The troubles stem in part from the same fact that makes LaTeX markup attractive for representing mathematical expressions in WordPress: it is extensively used in ordinary document preparation. This means that users were accustomed to exploiting the vast functionalities of LaTeX in its usual context without giving much thought to whether those particular functions would work in the same way or at all in the WordPress adaptation of the system—by no means a guarantee, even though the system in WordPress compiles mathematical expressions using a standard version of the LaTeX typesetting apparatus [17].[17] Moreover, the inability of users other than Gowers (or Tao, where his blog played host) to edit comments meant that the standard try-it-and-see approach to correcting typesetting errors offline was not available to most participants.[18]

In some cases, LaTeX markup was presented in a way such that it would be displayed without being rendered as a typeset mathematical expression [9]. There, users counted on common familiarity with the markup system to work around difficulties with that system's operation in WordPress. In other cases, what were intended to be ordinary, unproblematic plain-text inputs led to unexpected outputs in the published comments. This was a particular issue with a feature designed to facilitate displays of emotion in informal exchanges on WordPress blogs: the automatic parsing of 'smilies,' or text configurations meant to look like sideways faces, into icons showing those faces right-side-up. One participant followed-up his own comment by remarking: "oops, that cool-looking face is supposed to be a forty-eight (48)" [7, #48 et seq., 2 February].

5. CONCLUSIONS

The object of these last sections on technical adaptations is not to claim that navigating LaTeX markup or internal indexing was somehow pivotal to the structure of the mathematical project. It is, however, to suggest that these putatively procedural and technical concerns need to be made a part of a social account a project like Polymath1. Seen in the context of the project's goals and organization, these encounters and struggles with the medium itself played an important role in addressing the project's central challenge of marshalling a 'massive' pool of mathematical minds under a single stream of research. The work of managing inputs occasioned a strikingly large expenditure of thought, energy, and text in service of the ultimately mathematical work of the project.

The greatest lesson for would-be mathematical collaborators from Polymath1 is that, as Tao observed with respect to WordPress's commenting facility, the media of mathematical collaboration are flexible, but not without consequence. In a strict sense, it was not true that Gowers and his collaborators were 'free to make up conventions as we go along.' Everything that they did was constrained by their particular medium of interaction, as indeed it would be in any medium, blog or otherwise. Yet the blog medium they did use, especially when supplemented by other tools, proved remarkably adaptable. It seems unlikely that the project would have come to such a fortuitous conclusion had it played out strictly along the lines in which it began, but neither was it possible for such technical stasis to have been the case. To use a medium is both to adapt to it and to adapt it. To participate in a collaboration is to shape it.

When the WordPress platform was adapted for LaTeX, for instance, the intent of the adaptation could not have been for its use in a project like Polymath1. At the time, the paramount concern was likely the ability more clearly and effectively to present mathematical expressions in the form of known results or isolated curiosities. In Polymath1, however, WordPress LaTeX became an essential tool for putting mathematical ideas into play as they were taking shape—it was a frequent and sometimes temperamental presence in blog entry and comment alike, a necessary collaborator with whom every active user required an acquaintance.

Blogging platforms are designed with particular modes of flexibility in mind. They are highly adaptable to variations in content, reflecting the great variety of potential users of a single platform, but less adaptable to variations in structure, as the Polymath1 discussions regarding threading illustrate. Wiki platforms like the one used for the Polymath1Wiki, by comparison, have a great deal of flexibility in both structure and content, but for that very reason are less conducive to the sort of collaboration undertaken in Polymath1. First and foremost, the collaborators in the project seem to have required firm constraints in support of conversation-like modes of contribution in order to make the project's conceptual arc both manageable and accessible in a way oriented to its ongoing development.

Moreover, it was the willingness of contributors to devote their time and creativity to the project that may have proven most pivotal to the success of the medium. Later replicas of the pilot Polymath project have sailed or sunk according to their ability to sustain a critical level of participation from both their managers and contributors. While it is not possible to give a definitive measure of the effect of participant energy on an online collaboration, a measure which in any event is likely to depend heavily on the particular undertak-

[16]The converse situation, where markup languages for the Internet have been adapted for mathematical expressions, is also present. For instance, MathML is an attempt to adapt the ubiquitous html Internet markup language to mathematics communication [19].

[17]On the importance of platform interoperability in 'open community contribution systems' see [3].

[18]See Tao's remarks in [37]. Tao remarks, in particular, that "the LaTeX support included in the WordPress blog is valuable, even if it does act up sometimes." See also [7, 8 Febuary].

ing at hand, it can be said with confidence that no medium can design away the constant need for a usership not just using the medium as planned but actively adapting it to the always changing and often uncertain needs of their collective work.

In retrospect, the media of significant changes tend to become invisible. Even in the highly reflexive environment of the Polymath experiment, where a steady supply of new projects has forced ongoing assessment of the technical means of collaboration, such details are invariably among the first considerations omitted when insiders and outsiders alike look back on Polymath1. There are good reasons for this. Commentators are interested in the project's mathematical achievements, and in its promise as a model of collaboration among human mathematicians. The hidden collaborators in the software and networks are meant to be written out of comprehensive narratives.

So it is telling in studying Polymath1, with its preserved record of both mathematical and social negotiations, how prominently the medium plays in the midst of the mathematics. If, as Nielsen argues, the Internet allows scientists to fashion a new sort of collective short-term memory positioned to serve the progress of research and collaboration, then perhaps it is appropriate for sociologists to see the Internet similarly. For Nielsen, the Internet is a versatile storehouse of facts and thoughts; for me, it holds as well the human-technology interactions through which those data are sedimented.

The unusual features of the Polymath1 project made these interactions particularly visible. Its reflexive and deliberate use of blogging in a largely unprecedented way forced what Gowers called 'Questions of procedure' into far more parts of the project than Gowers perhaps intended.[19] It may turn out that Polymath1 was an aberration, that it will fade in the face of other modes, existing or yet-to-be-devised, of mathematical collaboration. But even if this is the case, it will have helped to shed light on the stubbornly elusive social character of users' attempts to manage and re-imagine the affordances of a software platform.

As a record of socio-technical organization in action, Polymath1's promise for sociologists ought to ring just as loud as its promise for mathematicians.

6. ACKNOWLEDGMENTS

This work has been funded by the Marshall Aid Commemoration Commission. I would like to thank Apostolos Doxiadis for making me aware of Polymath and for suggesting its fruitfulness for my own research interests.

7. REFERENCES

[1] Background to a polymath project. http://gowers.wordpress.com/2009/01/30/background-to-a-polymath-project/, January 2009.

[2] C. Borgman. *Scholarship in the Digital Age: Information, Infrastructure, and the Internet*. The MIT Press, Cambridge, Mass., 2007.

[3] N. Bos et al. From shared databases to communities of practice: A taxonomy of collaboratories. *Journal of Computer-Mediated Communication*, 12(2), 2007.

[4] G. C. Bowker. *Memory Practices in the Sciences*. The MIT Press, Cambridge, Mass., 2006.

[5] Brief review of polymath1. http://gowers.wordpress.com/2009/02/23/brief-review-of-polymath1/, February 2009.

[6] M. Castells. *The Internet Galaxy: Reflections on the Internet, Business, and Society*. Oxford University Press, Oxford, 2001.

[7] A combinatorial approach to density Hales-Jewett. http://gowers.wordpress.com/2009/02/01/a-combinatorial-approach-to-density-hales-jewett/, February 2009.

[8] DHJ—possible proof strategies. http://gowers.wordpress.com/2009/02/13/dhj-possible-proof-strategies/, February 2009.

[9] DHJ—quasirandomness and obstructions to uniformity. http://gowers.wordpress.com/2009/02/08/dhj-quasirandomness-and-obstructions-to-uniformity/, February 2009.

[10] DHJ—the triangle-removal approach. http://gowers.wordpress.com/2009/02/06/dhj-the-triangle-removal-approach/, February 2009.

[11] J. Dyer. A gentle introduction to the polymath project. http://numberwarrior.wordpress.com/2009/03/25/a-gentle-introduction-to-the-polymath-project/, March 2009.

[12] T. Gowers. Problems related to littlewood's conjecture. http://gowers.wordpress.com/2009/11/17/problems-related-to-littlewoods-conjecture-2/, November 2009.

[13] T. Gowers and M. Nielsen. Massively collaborative mathematics. *Nature*, 461(15):879–881, 2009.

[14] J. Grudin. Computer-supported cooperative work: History and focus. *IEEE Computer*, 27(5):19–26, 1994.

[15] Introspection. http://virtualteatime.blogspot.com/2009/02/introspection.html, February 2009.

[16] Is massively collaborative mathematics possible? http://gowers.wordpress.com/2009/01/27/is-massively-collaborative-mathematics-possible/, January 2009.

[17] Wordpress latex support. http://en.support.wordpress.com/latex/.

[18] B. Latour and S. Woolgar. *Laboratory Life: The Construction of Scientific Facts*. Princeton University Press, Princeton, NJ, 1986.

[19] R. Miner. The importance of mathml to mathematics communication. *Notices of the American Mathematical Society*, 52(5):532–538, 2005.

[20] Multidimensional noughts and crosses. *Nature*, 461(15):880.

[21] n-category café. http://golem.ph.utexas.edu/category/2008/02/metric_spaces.html.

[22] M. Nentwich. Cyberscience: The age of digitized collaboration? In G. Olson (et al.), editor, *Scientific Collaboration on the Internet*. The MIT Press, Cambridge, Mass., 2008.

[23] M. Nielsen. http:

[19] For attempts to cordon off such discussion, see [16]. The final rule under [33] begins with "Comments on the collaborative procedure should be carefully kept apart from the mathematical comments."

//michaelnielsen.org/blog/michael-a-nielsen/.

[24] M. Nielsen. The future of science. http://michaelnielsen.org/blog/the-future-of-science-2/, July 2008.

[25] M. Nielsen. Doing science online. http://michaelnielsen.org/blog/doing-science-online/, January 2009.

[26] M. Nielsen. Is massively collaborative mathematics possible? http://michaelnielsen.org/blog/is-massively-collaborative-mathematics-possible/, January 2009.

[27] M. Nielsen. The polymath project: scope of participation. http://michaelnielsen.org/blog/the-polymath-project-scope-of-participation/, March 2009.

[28] M. Nielsen. Update on the polymath project. http://michaelnielsen.org/blog/update-on-the-polymath-project/, February 2009.

[29] M. Nielsen. Introduction to the Polymath Project and "Density Hales-Jewett and Moser Numbers." http://michaelnielsen.org/blog/introduction-to-the-polymath-project-and-density-hales-jewett-and-moser-numbers/, May 2010.

[30] G. Olson (et al.). A theory of remote scientific collaboration. In G. Olson (et al.), editor, *Scientific Collaboration on the Internet*. The MIT Press, Cambridge, Mass., 2008.

[31] Polymath1wiki. http://michaelnielsen.org/polymath1/.

[32] Problem solved (probably). http://gowers.wordpress.com/2009/03/10/problem-solved-probably/, March 2009.

[33] Questions of procedure. http://gowers.wordpress.com/2009/02/01/questions-of-procedure/, February 2009.

[34] Quick question. http://gowers.wordpress.com/2009/02/04/quick-question/, February 2009.

[35] C. Rosental. *Weaving self-evidence: a sociology of logic*. Princeton University Press, Princeton, 2008.

[36] T. Tao. http://terrytao.wordpress.com/category/question/.

[37] T. Tao. IMO 2009 q6 mini-polymath project: impressions, reflections, analysis. http://terrytao.wordpress.com/2009/07/22/imo-2009-q6-mini-polymath-project-impressions-reflections-analysis/, July 2009.

[38] To thread or not to thread. http://gowers.wordpress.com/2009/02/21/to-thread-or-not-to-thread/, February 2009.

[39] Introduction to blogging: Trackbacks. http://codex.wordpress.org/Introduction_to_Blogging#Trackbacks.

[40] Upper and lower bounds for the density Hales-Jewett problem. http://terrytao.wordpress.com/2009/02/05/upper-and-lower-bounds-for-the-density-hales-jewett-problem/, February 2009.

[41] Why this particular problem? http://gowers.wordpress.com/2009/02/01/why-this-particular-problem/, February 2009.

[42] Wikipedia. http://en.wikipedia.org/wiki/Wikipedia.

[43] S. Woolgar. Laboratory studies: A comment on the state of the art. *Social Studies of Science*, 12(4):481–498, 1982.

[44] The year in ideas. http://www.nytimes.com/projects/magazine/ideas/2009/#m-2, 2009.