‘[B]ut this is blog maths and we’re free to make up conventions as we go along’

Polymath1 and the Modalities of ‘Massively Collaborative Mathematics’

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Outline

- Gowers Posed a problem and some ground rules

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- They solved the problem

The Polymath Project: What Happened

The Conventional Story

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- They solved a bigger problem
- They published a paper under the pseudonym DHJ Polymath

The Mathematical Problem

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The Socio-Technical Problem

- ‘Blog Maths’
- Identifying good contributions
- Publication, Credit

The Polymath Project: What Happened
Ground Rules

Comments

- Short
- Numerous
- Minimally Technical
- Constructive
- ‘Quantum of progress’

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The Project

- Pseudonymous Publication
- Subdiscussions get new posts, summaries
- Separate Procedure and Mathematics

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- Blog
- Posts
- Trackbacks
- Comments
- Sub-comments

The Polymath Project: The Medium

Blogs as Memory

Blogs are a form of memory (Nielsen)

Short-term working memory
Long-term archival memory

The Polymath Project: The Medium
Blog

The Polymath Project: The Medium

Posts

« Questions of procedure   Why this particular problem? »

A combinatorial approach to density Hales-Jewett

By gowers

Here then is the project that I hope it might be possible to carry out by means of a large collaboration in which no single person has to work all that hard (except perhaps when it comes to writing up). Let me begin by repeating a number of qualifications, just so that it is clear what the aim is.

1. It is *not* the case that the aim of the project is to find a combinatorial proof of the density Hales-Jewett theorem when $k = 3$. I would love it if that was the result, but the actual aim is more modest: it is *either* to prove that a certain approach to that theorem (which I shall soon explain) works, *or* to give a very convincing argument that that approach cannot work. (I shall have a few remarks later about what such a convincing argument might conceivably look like.)

2. I think that the chances of success even for this more modest aim

The Polymath Project: The Medium

Comment Form

The Polymath Project: The Medium

Trackbacks aand Comments

199 Responses to “A combinatorial approach to density Hales-Jewett”

A massively collaborative mathematical project « What’s new Says:
February 1, 2009 at 8:47 pm | Reply

[…] collaborative mathematical project” over at his blog. The project is entitled “A combinatorial approach to density Hales-Jewett”, and the aim is to see if progress can be made on this problem by many small contributions […]

gowers Says:
February 1, 2009 at 8:59 pm | Reply

1. A quick question. Furstenberg and Katznelson used the Carlson-Simpson theorem in their proof. Does anyone know that proof well enough to know whether the Carlson-Simpson theorem might play a role here? If so, I could add it to the background-knowledge post. (But I’m sort of hoping it won’t be needed.)

jozsef Says:

The Polymath Project: The Medium
Threading

Threading with Posts

- Summaries and Restatements
- Spectrum Division

Threading with Posts

- Summaries and Restatements
- Spectrum Division

Threading within Comments

- Disrupted temporal order
- Disrupted visual order
- Reinforced conceptual order

The Polymath Project: The Medium
Sub-comments

If we iterate this m times (replacing δ by $\varepsilon/2$ at each stage) we conclude that there exist disjoint wildcard sets $I_1, \ldots, I_m$ of size $O_{m,\delta}(1)$ such that the proportion of combinatorial m-spaces with these wildcards that completely lie in A is at least ε_m for some $\varepsilon_m = \varepsilon_m(\delta) > 0$.

This is the multidimensional DHJ(2). The standard derivation of DHJ(2.5) from DHJ(2) should then give the multidimensional DHJ(2.5) needed for my argument.

Terence Tao Says:
March 2, 2009 at 9:00 pm

The argument here, of course, is identical to the usual proof of the Szemeredi cube lemma (that dense subsets of [n] contain high-dimensional cubes), by first iterating the fact given that a subset A of [n] of density δ one can find a positive h such that $A' := (A + h) \cap A$ has density $\gg \delta^2$.

jozsef Says:
March 2, 2009 at 10:39 pm

Terry, I'm not sure that I understand what are you saying here. Would it lead to a Varnavides-type result? Here are the numbers that I see; If a set of subsets of [n] has at least $c_d 2^n / n^{1/2^d}$ elements, then it contains a d-dimensional subspace. This bound is close to be sharp. Using this we see that a c-dense subset of $2^{[n]}$ contains at least $cn^{1-1/2^d} 2^n$ d-dimensional subspaces, which is not much. Your calculation suggests much more.

LaTeX is. . .

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- (Mostly) operational in WordPress
- Ubiquitous
- Habitual

Methodological Challenges
Overview

- Temporal Issues
- Expert-knowledge and Access Issues
- Software Issues

Methodological Challenges
Temporal Issues

- Outcome already known
- ‘In the moment’ contextual details lost (mostly)
- Edits and amendments
- . . . but, there are many clues to aid reconstruction

Methodological Challenges
Expert-Knowledge and Access Issues

- Advanced mathematical training
- I'm neither Gowers nor Tao
- Can't see offline
- Software expertise
- ... but, expertise may be overrated

Methodological Challenges
Software Issues

- Revision history access
- Timestamp reliability
- Rendering issues can be edited-over
- Visual and functional stability

Theoretical Conclusions

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- Procedural and technical aspects should be an important part of social accounts

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