

‘[B]ut this is blog maths and we’re free to make  
up conventions as we go along’

Polymath1 and the Modalities of ‘Massively Collaborative  
Mathematics’

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# Outline

- 1 The Polymath Project
- 2 The Medium
- 3 Methodology
- 4 Conclusions

# The Polymath Project: What Happened

## The Conventional Story

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- They published a paper under the pseudonym DHJ Polymath

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- 'Multi-dimensional noughts and crosses' (tic-tac-toe)
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### The Socio-Technical Problem

- 'Blog Maths'
- Identifying good contributions
- Publication, Credit



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- Numerous
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### The Project

- Pseudonymous Publication
- Subdiscussions get new posts, summaries
- Separate Procedure and Mathematics

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# The Polymath Project: The Medium

## Overview

- Blog
- Posts
- Trackbacks
- Comments
- Sub-comments

# The Polymath Project: The Medium

Blogs as Memory

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## Blogs as Memory

Blogs are a form of memory (Nielsen)

Short-term working memory

Long-term archival memory

# The Polymath Project: The Medium

## Blog

The screenshot shows a web browser window displaying the homepage of Gowers's Weblog. The page has a blue header with the title "Gowers's Weblog" and the subtitle "Mathematics related discussions". Below the header, there is a main content area with a post titled "EDP16 - from AP-discrepancy to HAP-discrepancy?". The post is dated "July 4, 2010 by gowers". The text of the post discusses a strategy for proving EDP that I discussed in EDP15. To the right of the main content, there is a "Meta" section with links for "Register", "Log in", "Valid XHTML", "XFN", and "WordPress.com". Below the Meta section is a search box labeled "search this site". At the bottom of the page, there is a "Blogroll" section with links for "Absolutely useless" and "Emmanuel Kowaleki". The browser's address bar and navigation buttons are visible at the bottom of the window.

## Gowers's Weblog

Mathematics related discussions

### EDP16 — from AP-discrepancy to HAP-discrepancy?

July 4, 2010 by gowers

In this post I want to elaborate on a strategy for proving EDP that I discussed in EDP15. Briefly, the idea is to take a representation-of-identity proof of Roth's AP-discrepancy result and modify it so that it becomes a representation-of-diagonal proof of unbounded HAP-discrepancy.

The first step of this programme was an obvious one: obtain a clean and fully detailed proof in the APs case. That has now been completed, and a write-up can be found [here](#). For the benefit of anyone who is interested in

#### Meta

- » Register
- » Log in
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- » WordPress.com

#### Blogroll

- » Absolutely useless
- » Emmanuel Kowaleki

# The Polymath Project: The Medium

## Posts

« Questions of procedure

Why this particular problem? »

### A combinatorial approach to density Hales-Jewett

By gowers

Here then is the project that I hope it might be possible to carry out by means of a large collaboration in which no single person has to work all that hard (except perhaps when it comes to writing up). Let me begin by repeating a number of qualifications, just so that it is clear what the aim is.

1. It is *not* the case that the aim of the project is to find a combinatorial proof of the density Hales-Jewett theorem when  $k = 3$ . I would love it if that was the result, but the actual aim is more modest: it is *either* to prove that a certain approach to that theorem (which I shall soon explain) works, *or* to give a very convincing argument that that approach cannot work. (I shall have a few remarks later about what such a convincing argument might conceivably look like.)
2. I think that the chances of success even for this more modest aim

# The Polymath Project: The Medium

## Comment Form

### Leave a Reply

Name



E-mail (will not be published)

Website

Notify me of follow-up comments via email.

Subscribe to this site by email

Submit Comment

# The Polymath Project: The Medium

## Trackbacks and Comments

### 199 Responses to “A combinatorial approach to density Hales-Jewett”

#### **A massively collaborative mathematical project** « What's new Says:

February 1, 2009 at 8:47 pm | Reply

[...] collaborative mathematical project” over at his blog. The project is entitled “A combinatorial approach to density Hales-Jewett”, and the aim is to see if progress can be made on this problem by many small contributions [...]

#### **gowers** Says:

February 1, 2009 at 8:59 pm | Reply



1. A quick question. Furstenberg and Katznelson used the Carlson-Simpson theorem in their proof. Does anyone know that proof well enough to know whether the Carlson-Simpson theorem might play a role here? If so, I could add it to the background-knowledge post. (But I'm sort of hoping it won't be needed.)

#### **Jozsef** Says:

February 1, 2009 at 9:00 pm | Reply



# The Polymath Project: The Medium

## Threading



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### Threading with Posts

- Summaries and Restatements
- Spectrum Division

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### Threading within Comments

- Disrupted temporal order
- Disrupted visual order
- Reinforced conceptual order

# The Polymath Project: The Medium

## Sub-comments

If we iterate this  $m$  times (replacing  $\delta$  by  $\varepsilon/\mathcal{L}$  at each stage) we conclude that there exist disjoint wildcard sets  $I_1, \dots, I_m$  of size  $O_{m,\delta}(1)$  such that the proportion of combinatorial  $m$ -spaces with these wildcards that completely lie in  $A$  is at least  $\varepsilon_m$  for some  $\varepsilon_m = \varepsilon_m(\delta) > 0$ .

This is the multidimensional DHJ(2). The standard derivation of DHJ(2.5) from DHJ(2) should then give the multidimensional DHJ(2.5) needed for my argument.

### Terence Tao Says:

March 2, 2009 at 9:00 pm



The argument here, of course, is identical to the usual proof of the Szemerédi cube lemma (that dense subsets of  $[n]$  contain high-dimensional cubes), by first iterating the fact given that a subset  $A$  of  $[n]$  of density  $\delta$  one can find a positive  $h$  such that  $A' := (A + h) \cap A$  has density  $\gg \delta^2$ .

### jozsef Says:

March 2, 2009 at 10:39 pm



Terry, I'm not sure that I understand what are you saying here. Would it lead to a Varnavides-type result? Here are the numbers that I see; If a set of subsets of  $[n]$  has at least  $c_d 2^n / n^{1/2^d}$  elements, then it contains a  $d$ -dimensional subspace. This bound is close to be sharp. Using this we see that a  $c$ -dense subset of  $\mathcal{2}^{[n]}$  contains at least  $c n^{1-1/2^d} 2^n$   $d$ -dimensional subspaces, which is not much. Your calculation suggests much more.

# The Polymath Project: The Medium

Embedded  $\text{\LaTeX}$  Mathematics

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Embedded  $\text{\LaTeX}$  Mathematics

$\text{\LaTeX}$  is...

- (Mostly) operational in WordPress
- Ubiquitous
- Habitual

# Methodological Challenges

## Overview

- Temporal Issues
- Expert-knowledge and Access Issues
- Software Issues

# Methodological Challenges

## Temporal Issues

- Outcome already known
- 'In the moment' contextual details lost (mostly)
- Edits and amendments
- ... but, there are many clues to aid reconstruction

# Methodological Challenges

## Expert-Knowledge and Access Issues

- Advanced mathematical training
- I'm neither Gowers nor Tao
- Can't see offline
- Software expertise
- ... but, expertise may be overrated



# Methodological Challenges

## Software Issues

- Revision history access
- Timestamp reliability
- Rendering issues can be edited-over
- Visual and functional stability

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- Communication leaves traces of how it is done
- Procedural and technical aspects should be an important part of social accounts

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- The relevant constraints and possibilities need not correspond to the designed-for ones
- It helps to have active participants